A New Type of Twin Prime

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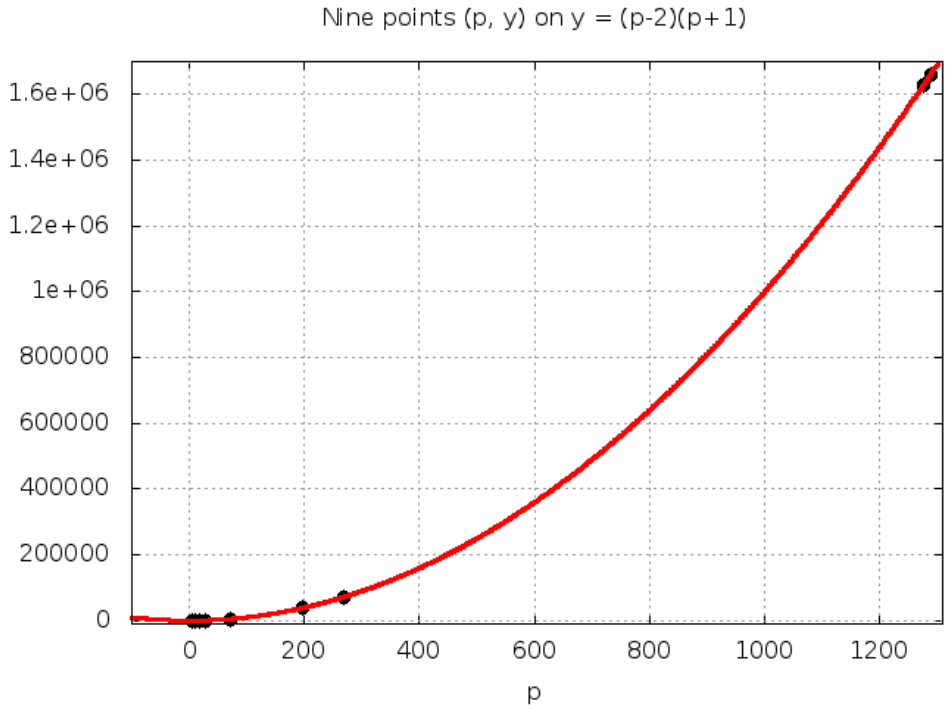
This article demonstrates that larger twin prime pairs can be obtained by plugging smaller ones into a quadratic equation. Specifically, we consider the sequence of lesser twin primes p, such that $p^2 - (p + 2)$ is the average of a larger twin prime pair $(y - 1,\ y + 1)$. Or:

$$y = p^2 - (p + 2)$$

Not all y are averages of larger twin prime pairs. To download a file listing the first 10,000 p and y: np-results-10k.txt.



Below is a table of the first 28 solutions. The twin prime 3 is not included, because when plugged into the quadratic, it returns 4, which is a twin prime average, but is equal to (not *larger than*) the average of (3, 5):

for $y = p^2 - (p + 2)$

n	p	y	n	p	y
1	5	18	15	3467	12016620
2	11	108	16	4649	21608550
3	17	270	17	6359	40430520
4	29	810	18	6761	45704358
5	71	4968	19	6827	46601100
6	197	38610	20	7877	62039250
7	269	72090	21	9461	89501058
8	1277	1629450	22	10529	110849310
9	1289	1660230	23	12917	166835970
10	1607	2580840	24	13337	177862230
11	2027	4106700	25	13691	187429788
12	2111	4454208	26	13829	191227410
13	2267	5137020	27	13931	194058828
14	2687	7217280	28	17291	298961388

All p ending in 7 or 9 produce a y ending in 0.
All p ending in 1 produce a y ending in 8. This can be understood by examples:

$17^2 - 17 - 2 = 270$. Ends in 0, like all y from p ending in 7

$29^2 - 29 - 2 = 810$. Ends in 0, like all y from p ending in 9

$11^2 - 11 - 2 = 108$. Ends in 8, like all y from p ending in 1

Here is a *maxima* program that generates n, p, y. Replace (suitably large integer) with a suitably large integer (without the parentheses):

```
y:0$ p:0$ c:0$ f(p):= p^2-p-2$
for p:5 thru (suitably large integer) step 6 do
(if(primep(p) and primep(p+2)) then
(y:f(p), if(primep(y-1) and primep(y+1)) then
(c:c+1, print(c,", ",p,", ", y)))) /* Michael Kaarhus */
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