Errors in Defining Prime

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Abstract

This article points out errors in the way some people define *prime number*.

In my investigations into twin primes, I encounter various definitions of *prime number*. Here is a typical one:

> The number p is *prime* if p is a positive integer that has exactly two unique factors: $\{1 \text{ and } p\}$.

The above definition is vague. It can actually mean three different things, and two of those meanings are false! Here they are:

- Meaning 1: p is prime if p is a positive integer that has exactly two unique factors: $\{1 \text{ and } p\}$.

The above meaning is false! Each prime p has exactly four unique factors: $\{-p, -1, 1, \text{ and } p\}$. It's true that $-p$ and -1 are not prime. However, they are unique factors of p.

- Meaning 2: p is prime if p is a positive integer that has exactly two unique *prime* factors: $\{1 \text{ and } p\}$.

The above meaning is false! Each prime p has exactly *one* unique prime factor: p. It's true that 1 is a unique factor of p. However, 1 is not prime; 1 has only *one* unique positive factor (not two, as primes do). Also, a good definition does not invoke the term it is defining (*prime* in this case).

- Meaning 3: p is prime if p is a positive integer that has exactly two unique *positive* factors: $\{1 \text{ and } p\}$.

Of the above, only Meaning 3 is a correct definition. Some might think it would be correct to say:

- p is prime if p is a positive integer that has exactly one unique prime factor.

But that isn't correct, either. The primes share that definition with other kinds of numbers. For instance, the squares of primes have exactly one unique prime factor – one that is used twice. For instance, the unique factors of 169 are {-169, -13, -1, 1, 13, 169}. Of those, 13 is the only prime factor; even though it is applied twice, it is only one number.