Twin Prime Quads Family II

Michael G. Kaarhus

May 21, 2013

Abstract

This article presents a second family of quadratics that produces central composites (averages) of twin prime pairs. Supplies more evidence that the twin primes are infinitely many.

If one twin prime pair-generating family of quadratics was not enough for you, here's another:

(1) $$y = \frac{(x-h)\cdot(x+h+3)}{2} \quad \text{where } h = \{1, 2, 3, 4, \ldots \infty\}, \quad h = \{-4, -5, -6, -7, \ldots -\infty\}.$$

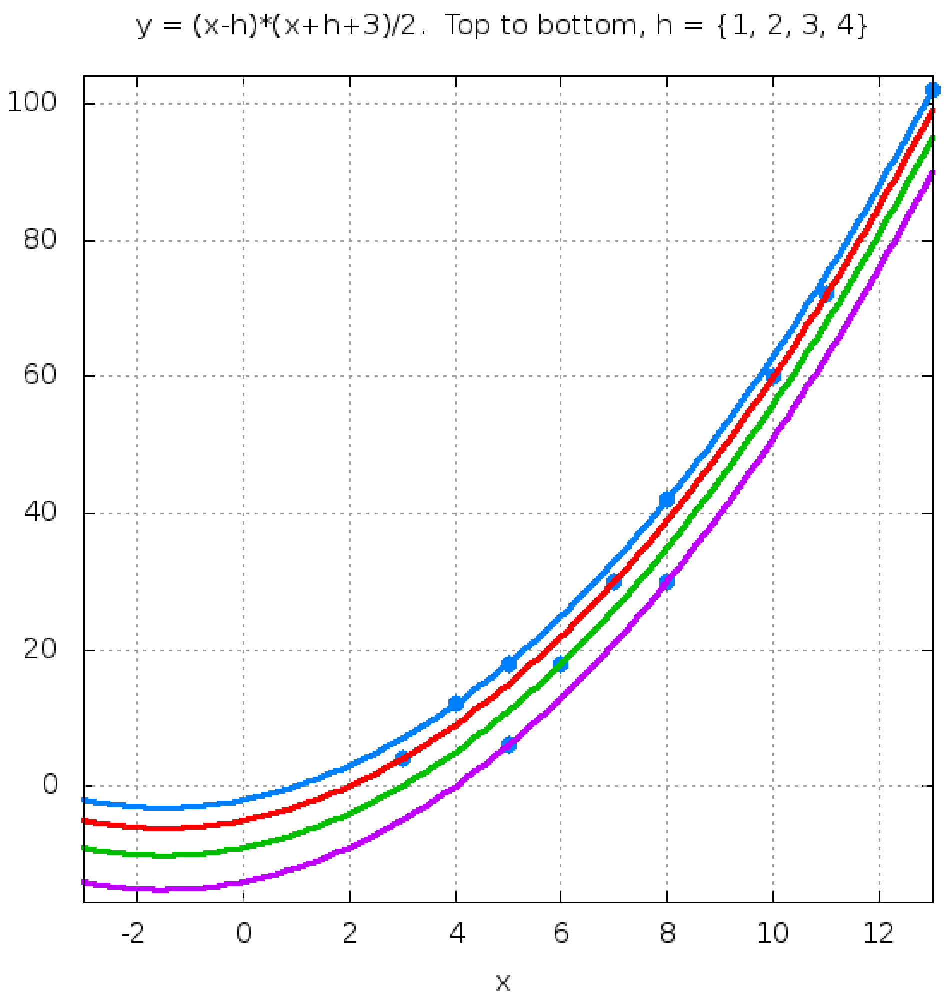
Some examples:

$$y = \frac{(x-1)\cdot(x+4)}{2}$$
$$y = \frac{(x-2)\cdot(x+5)}{2}$$
$$y = \frac{(x-3)\cdot(x+6)}{2} \quad \text{etc.}$$

Stats for $y = \frac{(x-h)\cdot(x+h+3)}{2}$

$y =$	**Yield of centrals for** $x = 1$ **to** $10,000$
$(x^2+3x-4)/2$	324 centrals
$(x^2+3x-10)/2$	184 centrals
$(x^2+3x-18)/2$	111 centrals
$(x^2+3x-28)/2$	144 centrals
$(x^2+3x-40)/2$	129 centrals
$(x^2+3x-54)/2$	83 centrals
$(x^2+3x-70)/2$	150 centrals
$(x^2+3x-88)/2$	175 centrals
$(x^2+3x-108)/2$	58 centrals
$(x^2+3x-130)/2$	134 centrals
$(x^2+3x-154)/2$	170 centrals

Below is the domain near $x = -\frac{3}{2}$. Centrals appear as dots. Centrals from negative x are omitted, as are negatives of centrals. These quads intercept the x-axis at adjacent, integral x.



There is a family of central-producing quads here, which are identical, except for their vertical shifts (which are determined by h). Each successive integer h shifts the parabola downward, and one integer to the right at the +x intercept. That is because the $(x-h)$ term zeros the function when $x = h$.

For the negative x that produce the same y values (and the same centrals) as the non-negative

x, we have:

$$
\begin{aligned}
f(x) &= \frac{x^2 + 3x - h^2 - 3h}{2} \\
f(-x) &= \frac{x^2 - 3x - h^2 - 3h}{2} \\
f(x-3) &= \frac{(x-3)^2 + 3(x-3) - h^2 - 3h}{2} \\
&= \frac{x^2 - 6x + 9 + 3x - 9 - h^2 - 3h}{2} \\
&= \frac{x^2 - 3x - h^2 - 3h}{2} \\
\therefore f(x-3) &= f(-x)
\end{aligned}
$$

The negative x that produce the same centrals are, in absolute value, three more than the positive x. For instance, if $f(-9)$ produces a central, then $f(6)$ produces the same central.

All quads in this family extend upward, and are nested. To find the line containing their vertices,

$$
\begin{aligned}
\frac{d}{dx}\left(\frac{x^2 + 3x - h^2 - 3h}{2}\right) &= x + \frac{3}{2} \\
x + \frac{3}{2} &= 0 \\
x &= -\frac{3}{2}
\end{aligned}
$$

I observe that, when $h = 1$, all the x that yield centrals are $\equiv \{1, 4, 5, 8\} \pmod{12}$. You can use that fact to sieve the input to the function, and check only $1/3$ of the integers you would otherwise have to check.

The first 30 (x, y) **for** $y = (x^2 + 3x - 4)/2$

n	(x, y)	n	(x, y)
1	(4, 12)	16	(76, 3000)
2	(5, 18)	17	(88, 4002)
3	(8, 42)	18	(89, 4092)
4	(13, 102)	19	(113, 6552)
5	(16, 150)	20	(133, 9042)
6	(20, 228)	21	(140, 10008)
7	(25, 348)	22	(148, 11172)
8	(28, 432)	23	(152, 11778)
9	(29, 462)	24	(164, 13692)
10	(40, 858)	25	(181, 16650)
11	(44, 1032)	26	(185, 17388)
12	(52, 1428)	27	(193, 18912)
13	(53, 1482)	28	(197, 19698)
14	(61, 1950)	29	(205, 21318)
15	(64, 2142)	30	(241, 29400)

For a list of the first 8,018 (x, y), download this file: fam2-h1.txt.

Some quads duplicate centrals in other quads. But the sequence of centrals each quad produces is apparently unique. As with the previous family, if each quad in family (1) produces a unique sequence of centrals (even though some quads duplicate some centrals generated by other quads), and if there are infinitely many of these quads (as there apparently are), then the twin primes are infinitely many.

Copyright